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Frictional shear cracks

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We discuss crack propagation along the interface between two dissimilar materials. The crack edge separates two states of the interface, "stick" and "slip". In the slip region we assume that the shear stress is proportional to the sliding velocity, i.e. the linear viscous friction law. In this picture the static friction appears as the Griffith threshold for crack propagation. We calculate the crack velocity as a function of the applied shear stress and find that the main dissipation comes from the macroscopic region and is mainly due to the friction at the interface. The relevance of our results to recent experiments, Baumberger et al., Phys. Rev. Lett. **88**, 075509 (2002), is discussed.

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A few recent experimental observations of Rubio and Galeano [1], and Baumberger, Caroli, and Ronsin [2], on the frictional motion of sheared gels sliding along a glass surface indicate the existence of self-healing pulses and inhomogeneous modes of sliding [3]. A regime of periodic stick slip has been observed in a limited range of small shearing rates [2]. It bifurcates towards stationary sliding at some critical driving velocity. The slip pulses traverse the sample with a velocity much larger than the driving velocity but still much smaller than the speed of sound.

Slip pulses in gels seem to be very different from Schallamach waves and "brittle" pulses studied by Gerde and Marder [4] since no observable interface separation occurs. In this respect, they are more comparable with self-healing cracks suggested by Heaton [5] in the context of seismic events.

Recent investigations (see, for example, [6] and references therein) point towards an essential importance of the underlying friction law in the slip state. It has been proved that the simple Coulomb friction leads to the so-called "ill-posedness" of the linear stability problem while discussing small nonhomogeneous perturbations of the stress and strain fields in a sliding mode [6]. Moreover, Caroli [7] has shown that the existence of slow, periodic slip pulses is incompatible with Coulomb friction law.

In this letter we discuss crack propagation along the interface between two dissimilar materials. The crack edge separates two states of the interface, "stick" and "slip". We assume that the interface is flat with a strong adhesion contact. In principle, we could allow for small wavelength surface roughness, but in this case we consider length scales larger than the longest wavelength component. In the presence of roughness, the assumption of strong adhesion and full contact at the interface presumably is only reasonable for "soft" materials with relatively small shear modulus. Gels are clearly materials of this sort.

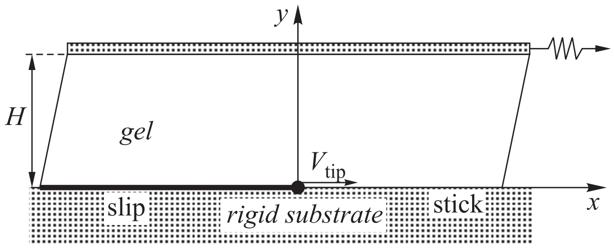
In the slip region we assume a simple linear viscous friction law, namely, the shear stress is proportional to the sliding velocity. This from the theoretical point of view strongly motivated law is usually not discussed in literature since it does not lead to the so-called static friction phenomenon observed experimentally. However, we will see that in our description static friction appears in a natural way as the usual Griffith threshold for crack propagation. The important point is that before the system goes into a sliding mode the slip pulse should traverse the sample. This requires finite shear stress since the stick state of the interface is energetically more favorable.

With the linear viscous friction law we find conditions for crack propagation and calculate the crack velocity as a function of an applied shear stress. We find that the main dissipation comes from the macroscopic region and is due to the friction at the interface. This situation is very different from the usual crack propagation where the main dissipation is localized in the microscopic tip region.

We also shortly discuss frictional shear cracks inside homogeneous materials. The point here is that in mode II (and in mode III) cracks there is no macroscopic opening. If two surfaces remain in contact, the standard boundary conditions, namely the vanishing of normal and shear stresses on the crack surfaces, are not theoretically motivated. The relative sliding velocity of the two surfaces should lead to nonzero shear stresses.

Finally we discuss the relevance of our results to the experimental observations [2].

Consider an elastic solid sliding on a flat rigid substrate. Assume that the elastic solid occupies the space $H > y > 0$, and let (x, y, z) be a coordinate system with the plane $y = 0$ corresponding to the surface of the solid, see Figure. We discuss the plane strain situation



An elastic body sliding on a rigid substrate

with $u_z = 0$, where $\mathbf{u}$ is the displacement vector. We assume that the interface can be in two states: "stick" and "slip". The boundary between these two states is described by the crack edge which moves with a velocity V_{tip} in the x direction. In the stick region the displacements are continuous and since we assume a rigid substrate, the boundary conditions are: $u_x = u_y = 0$ for $x - V_{tip}t > 0$ and $y = 0$. In the slip region we assume that the two solids (for all times) are in contact, $u_y = 0$ for $x - V_{tip}t < 0$ and $y = 0$, while we allow for a finite relative sliding velocity $\dot{u}_x$. This sliding velocity leads to frictional shear stress at the interface where we assume a linear viscous friction law:

$$\sigma_{xy} = \alpha \dot{u}_x, \tag{1}$$

with α being the viscous friction coefficient. It is reasonable to assume that the interface energy in the stick phase is smaller than the interface energy in the slip phase since the adhesion contact in the stick region is stronger. Let us denote this energy difference by γ. It is clear that without external loading the stick phase is energetically favorable and a finite shear stress is required to get the interface into the slip state. Let us assume that far ahead of the crack tip, the solid is homogeneously strained with u_{xy}^{∞} and stressed with $\sigma_{xy} = 2\mu u_{xy}^{\infty}$ where μ is the shear modulus. The strain energy is $\mu(u_{xy}^{\infty})^2 H$. Far behind the crack tip where the stress is relaxed, only the interface energy γ remains. The slip state will be realized only if $\Delta = \mu(u_{xy}^{\infty})^2 H/\gamma > 1$. In this case the crack should propagate in the positive x direction. Otherwise, the crack would propagate with negative velocity and the stick phase will be restored. Condition $\Delta = 1$ is nothing but the usual Griffith threshold for crack propagation. On the other hand, in the context of the friction problem, this condition may be interpreted as a static friction threshold: a finite shear loading is required to get the system into the sliding mode.

If the whole interface is in the slip state, a steady-state motion of the elastic body is possible with a velocity

$$V = 2Su_{xy}, \tag{2}$$

where $S = \mu/\alpha$ is the velocity scale given by the friction law. We note that this homogeneous sliding mode is linearly stable for any velocity with respect to small nonhomogeneous perturbations of the stress and strain fields localized in the surface region. In this respect the viscous friction law is very different from Coulomb friction which leads to a linear instability and ill-posedness of the problem as it has been intensively discussed in literature [6].

On the other hand, the homogeneous sliding mode may be unstable against the resticking pulse (nonlinear "healing instability") if the corresponding value of $\Delta < 1$. Since in this case, the strain which defines the value of Δ is related to the steady-state sliding velocity by Eq.(2), we find that the homogeneous sliding is stable against the healing instability only above the critical sliding velocity

$$V_c = 2S(\gamma/\mu H)^{1/2}. \tag{3}$$

Now let us turn to the calculation of the crack tip velocity V_{tip} as a function of the dimensionless driving force Δ. The strategy is as follows: we solve the elastic problem in the vicinity of the crack tip and then calculate the energy flux into the crack tip and the dissipation due to the friction at the interface. Finally, using the energy balance, we find the crack velocity.

Let us start from some qualitative estimates. Assume that, as in the usual crack problem, the singular behavior of the displacement vector is given by a square-root singularity in the vicinity of the crack tip. Then the dissipation rate at the interface

$$J_d = \alpha \int (\dot{u}_x)^2 dx \tag{4}$$

depends logarithmically ($\sim \ln H/a$) on H. Thus, the main dissipation comes from the macroscopic region. In the usual crack problem the main dissipation comes from the close vicinity of the tip and often requires the introduction of microscopic models. Here we have the chance to avoid such a detailed microscopic description by using some microscopic length scale a as a cutoff which enters only the logarithm in the final result. In this small region the used equations break down and an effective tip dissipation should be introduced. Due to bulk viscosity there is an additional contribution $\eta\dot{u}_{ik}$ to the stress tensor. The dissipation rate diverges strongly at small distances (as $1/r$) and correspondingly decays at macroscopic distances. Thus, this effect can be also incorporated into the tip dissipation.

Now we solve the elastic problem more accurately, while still using a quasistatic approximation for the moment. The generalization to the full elastodynamic description is straightforward and will be given below. In the co-moving frame of reference and in the vicinity of the crack tip, the displacement field for the static elasticity and the boundary conditions formulated above is

$$u_x = A\,\mathrm{Re}\left[yz^{\lambda-1} - i\,(3-4\nu)\,z^{\lambda}/\lambda\right],$$

$$u_y = A\,\mathrm{Re}\left[iyz^{\lambda-1}\right]. \qquad (5)$$

Here z is a complex coordinate $z = x+iy$, ν is the Poisson ratio, and A is a real amplitude. The spectrum of λ is purely real and given by the following equation:

$$\exp\left(i2\pi\lambda\right) = -\frac{1+i\pi\varepsilon/2}{1-i\pi\varepsilon/2}, \qquad (6)$$

with

$$\varepsilon = \frac{1}{2\pi}\frac{3-4\nu}{1-\nu}\frac{V_{tip}}{S}. \qquad (7)$$

In the limit of small values of $|\varepsilon|$ for the leading crack displacement component, we have $2\lambda \approx 1+\varepsilon$.

Having defined the displacement field we can calculate the energy flux $J_i = \sigma_{ik}\dot{u}_k$ and the local energy release into a small semicircular region with some microscopic radius a around the crack tip,

$$J_0 = 2\pi\mu(3-4\nu)(1-\nu)V_{tip}A^2a^{\varepsilon}. \qquad (8)$$

The dissipation rate due to the interface friction with exclusion of the small region of size a close to the tip is given by Eq. (4):

$$J_d = \alpha(3-4\nu)^2A^2V_{tip}^2\varepsilon^{-1}\left(\widetilde{H}^{\varepsilon} - a^{\varepsilon}\right) = J_0\left[(\widetilde{H}/a)^{\varepsilon} - 1\right]. \qquad (9)$$

Here $\widetilde{H} = f(\nu)H$, with f being an undetermined yet function of the Poisson ratio; H is a thickness of the sample. This function can be found by solving the elastic problem for given geometry and all boundary conditions. As we will see the tip velocity does not crucially depends on the actual value of the factor f, which is of the order of unity.

On the other hand the local energy release into the crack tip J_0 must compensate the surface energy difference: $J_0 = \gamma V_{tip}$. Note, that here we have neglected the dissipation at the tip in comparison with the energy release γV_{tip}, which is reasonable at small tip velocities compared to the velocity of sound. Finally, using the global energy conservation law,

$$J_{\infty} = J_0 + J_d = \mu\left(u_{xy}^{\infty}\right)^2 HV_{tip}, \qquad (10)$$

we find

$$\varepsilon = \frac{\ln\Delta}{\ln(\widetilde{H}/a)} \approx \frac{\ln\Delta}{\ln(H/a)}. \qquad (11)$$

Since ε is given by Eq. (7), this result is a compact representation of the crack velocity as a function of the driving force $\Delta = \mu\left(u_{xy}^{\infty}\right)^2 H/\gamma$. Note that $\Delta = 1$ corresponds to Griffith equilibrium. Eq. (11) is valid for small $|\varepsilon|$. The explicit expression for the crack velocity reads

$$V_{tip} = 2\pi\frac{1-\nu}{3-4\nu}\frac{\ln\Delta}{\ln(H/a)}S. \qquad (12)$$

In the case of small $\Delta - 1$ we obtain

$$V_{tip} \approx 2\pi\frac{1-\nu}{3-4\nu}\frac{\Delta-1}{\ln(H/a)}S. \qquad (13)$$

This result corresponds to a small dissipation rate compared to the total energy flux, $J_d \ll J_{\infty}$, and can also be obtained using perturbation theory: we solve the elastic problem neglecting friction ($\sigma_{xy} = 0$ at the interface) and then calculate the dissipation rate (4) using this solution as zero order displacement field.

Up to now we have used the static approximation. The tip velocity should be small compared to the sound velocity. The elastodynamic generalization is straightforward. Using the standard approach to the singular solutions near the crack tip [8], we find the displacement field $\mathbf{u}$

$$u_x = A\,\mathrm{Re}\left[\frac{(x+i\alpha_d y)^{\lambda}}{i\alpha_d} + i\alpha_s(x+i\alpha_s y)^{\lambda}\right],$$

$$u_y = A\,\mathrm{Re}\left[(x+i\alpha_d y)^{\lambda} - (x+i\alpha_s y)^{\lambda}\right] \qquad (14)$$

with $\alpha_d^2 = 1-(V_{tip}/C_d)^2$ and $\alpha_s^2 = 1-(V_{tip}/C_s)^2$, where C_d and C_s are dilation and shear wave speed. The spectrum of λ is still given by Eq. (6) but now ε reads

$$\varepsilon = \frac{2}{\pi}\frac{(1-\alpha_d\alpha_s)}{\alpha_d(1-\alpha_s^2)}\frac{V_{tip}}{S}. \qquad (15)$$

Thus, elastodynamic effects just lead to a redefinition of ε in the main result, Eq. (11), which remains valid. For small velocities, Eqs. (14) and (15) reduce to Eqs.(5) and (7), respectively.

The most serious problem with large velocities arises in connection with a self-consistent description of the dissipation at the tip. This part of kinetics cannot be considered macroscopically for arbitrary tip velocities. One can only treat the case of small velocities, $V_{tip} \ll C_s$ in a model independent way, by introducing the tip kinetic coefficient. For higher velocities the so-called velocity dependent fracture energy $\gamma(V_{tip})$ is introduced. This function contains information about the usual surface energy γ and tip dissipation and reduces to the surface energy in the static limit. The main dissipation in our approach arises from the friction between both sides of the crack and can be treated macroscopically. Note that this part of the dissipation can be described by the velocity independent friction coefficient α even at large tip velocities in the case of small sliding velocities.

Up to now we have discussed shear cracks along the interface between two dissimilar materials (the case of a rigid substrate). We also present the result for the case of frictional shear cracks inside homogeneous elastic materials. Such cracks can propagate under the shear loading in amorphous materials, or along grain boundaries in crystals (in single crystals the well known dislocation mechanism of plasticity should be favorable). The boundary conditions on the crack surfaces, which remain in contact, are: continuous normal displacement and continuous normal and shear stresses. The shear stress is also given by Eq.(1) with $\dot{u}_x$ being the relative sliding velocity of two crack surfaces. We note that these boundary conditions are quite different from the standard boundary conditions of mode II (and III) cracks: zero normal and shear stresses on the crack surfaces [8]. In our case, the frictional shear crack, we find the following expression for ε which enters the general result, Eq.(11):

$$\varepsilon = \frac{4}{\pi}\frac{\alpha_s(1-\alpha_s^2)}{4\alpha_d\alpha_s-(1+\alpha_s^2)^2}\frac{V_{tip}}{S}. \tag{16}$$

Now let us discuss the relation of our results to experimental observations of Baumberger, Caroli and Ronsin [2]. They performed experiments of a gel sliding on a glass plate. The driving velocity was given and the shear force and thus the average shear stress was deduced from the spring elongation. Above some critical driving velocity $V_c \approx 125\,\mu$m/s, steady sliding was observed. At velocities smaller than the critical one, periodic stick slip sets up (see figures in [2]). Upon increasing the driving velocity V, no hysteresis of the transition was detected. In the stick slip regime they observed the propagation of self-healing pulses with no opening, nucleated periodically at the trailing edge of the sample. The propagation velocity of these cracks was about 60 times larger than the critical sliding velocity, yet still much smaller than the shear wave speed.

The existence of a critical sliding velocity, where stationary sliding is stable against the healing instability, appears naturally in our description and is given by Eq. (3). The characteristic value of the shear strain in the sliding mode near the critical velocity experimentally was about $u_{xy} = 0.04$. Thus, we can estimate from Eq. (2) the characteristic velocity $S = 1.5\,$mm/s and from Eq. (3) we find that the characteristic difference between the interface energies in the slip and stick states is $\gamma = 0.1\,\mathrm{J/m^2}$. One would expect that for ordinary elastic materials the velocity S should be of the order of the speed of sound. However, for gels the shear modulus μ is much smaller than for ordinary materials. The shear wave speed $C_s = (\mu/\rho)^{1/2}$ is only $2\,$m/s. The velocity $S = \mu/\alpha$ is linear in μ and should be even smaller. This is a possible explanation for a relatively small value of S compared to C_s.

In the stick-slip regime, which exists below V_c, the nucleation of a slip pulse takes place at the trailing edge of the sample and requires overshooting above the Griffith threshold according to the experiment. This overshooting is not small and in order to estimate the crack velocity we can use Eq. (12) since the velocity is still much smaller than the speed of sound. Because of the week logarithmic parameter dependence, we conclude that V_{tip} is of the order of S and essentially independent of the driving velocity in agreement with experimental observations. After the slip pulse traversed the sample the stress drops below the Griffith threshold and resticking takes place via propagation of a healing pulse. Its velocity is still described by Eq. (12) with $\Delta < 1$. This periodic stick slip regime bifurcates towards stationary sliding at $V = V_c$ where stresses are always above the Griffith threshold. For driving velocities slightly below V_c, characteristic values of Δ for resticking are close to 1. Since the velocity of the resticking crack is small and comparable with the driving velocity in this range, a complicated collective behavior of self-healing pulses is observed [2].

While our results are in a qualitative agreement with experimental observations, we still underestimate the crack velocity which is a few times smaller than in the experiment. On the other hand, the geometry of the experiment is such that the total macroscopic friction of the sliding sample obviously depends on the processes taking place at the edges of the sample. The stresses here are highly inhomogeneous and the kinetic phenomena should be considered with a great care.

The observed nonlinear behavior of the stress with the velocity for relatively high sliding rates (the so-called shear-thinning rheology) can be in principle incorporated into a more sophisticated version of theory presented here. Another subject of future investigations should be the collective behavior of self-healing pulses in the spirit of Ref. [7]. Further theoretical and experimental investigations are needed to shed light on this phenomenon where two intriguing problems, crack propagation and friction, combine together.

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